Nonadiabatic quantum state manipulation of superconducting structures

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Why quantum computers?

Chip miniaturization

Hard computational problems

QC

Shrinking electronics

Electronics have been getting smaller and faster

- Moore's law: computer performance doubles every 18 months

- atomic levels will be reached in ?? Years

R. Feynmann (1985):

"it seems that the laws of physics
present no barrier to reducing
the size of computers until bits are
the size of atoms, and quantum
behavior holds sway"

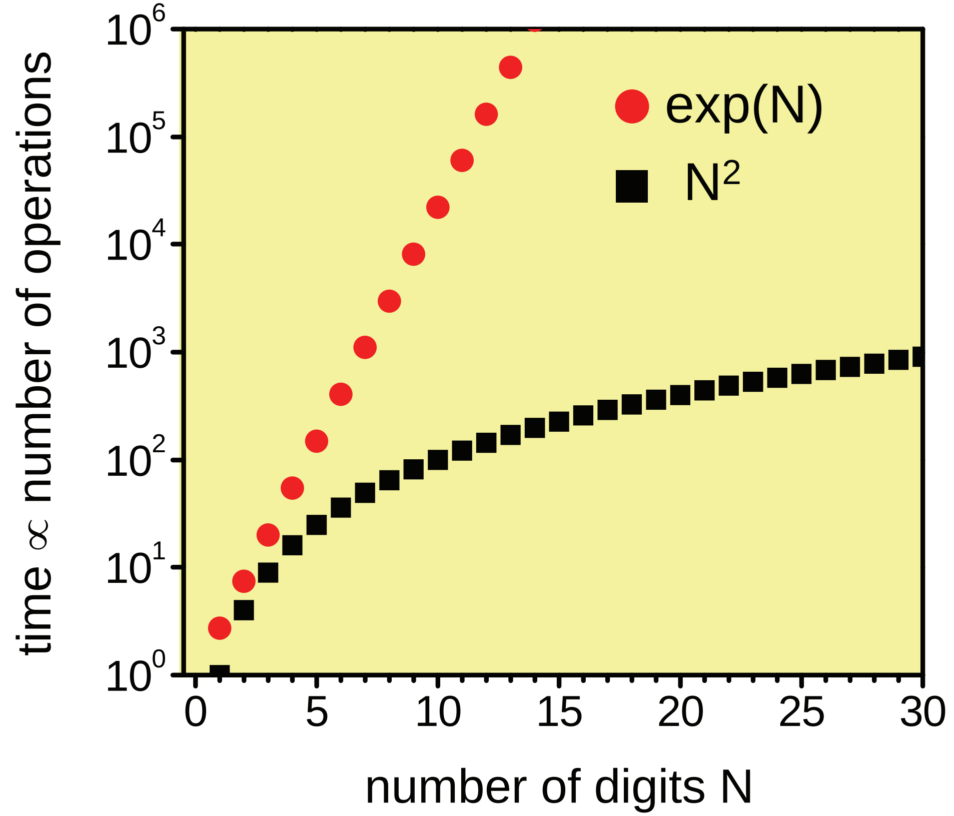
Complexity of computational problems

Complexity of a problem can be:
(N = number of digits in input)

polynomial $N_{op} \propto N^{a}$

non-polynomial $N_{op} \propto \exp(N)$

Computational time is proportional to the number of operations



Ex: Factorization of an integer in prime factors is NP problem
We know how to solve the problem but we do not have time!

Power of QC

Factorization of large integers (N digits)

<u>Classical algorithms:</u>

N = 129	8 months (1994)
N = 250	10^6 years (estimated)
N = 1000	10^{25} years (estimated)

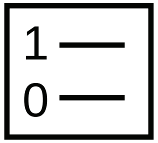
<u>Quantum algorithm (P. Shor, 1994):</u>

N = 1000	a few seconds

Hard computational problems

- factorization of large numbers --> Shor's algoritm
- database search --> Grover's algorithm
- graph isomorphism
- travelling salesman
- tailor's problem

Main features of QC

qu-bits: 

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$|\alpha|^2 + |\beta|^2 = 1$$

$|0\rangle$ with probability $|\alpha|^2$

$|1\rangle$ with probability $|\beta|^2$

qu-registers: a set of qubits, N qubits --> 2^N states

qu-gates: input - superposition of states

N - bit register represents 2^N states --> quantum parallelism

Any operation is performed on all the states at the same time --> exponential amount of computational space with linear amount of physical space

To build a QC we need:
(DiVincenzo criteria)

scalable qubits = two-level quantum systems

initialization = prepare qubit in a given state

state manipulation

read-out

quantum logic gates

"long" coherence time ($\tau >> 1/f_{clock}$)

Two-level systems

non solid-state

- atoms
- ions
- nuclear spins
- photons
- . . .

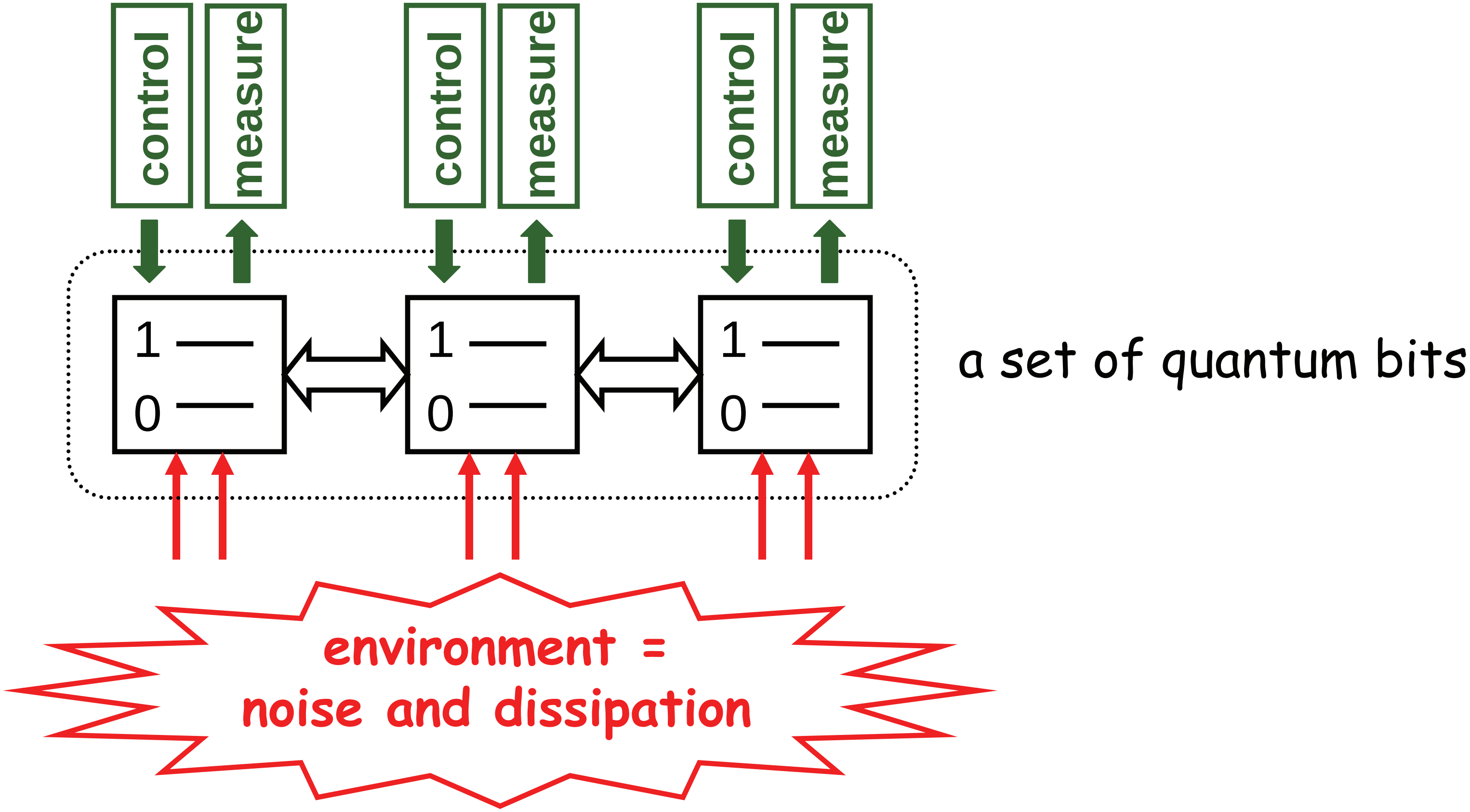
solid-state

- CP box ← this work
- RF SQUID
- 3J SQUID
- single J junction
- quantum dots
- . . .

coherence easier ←

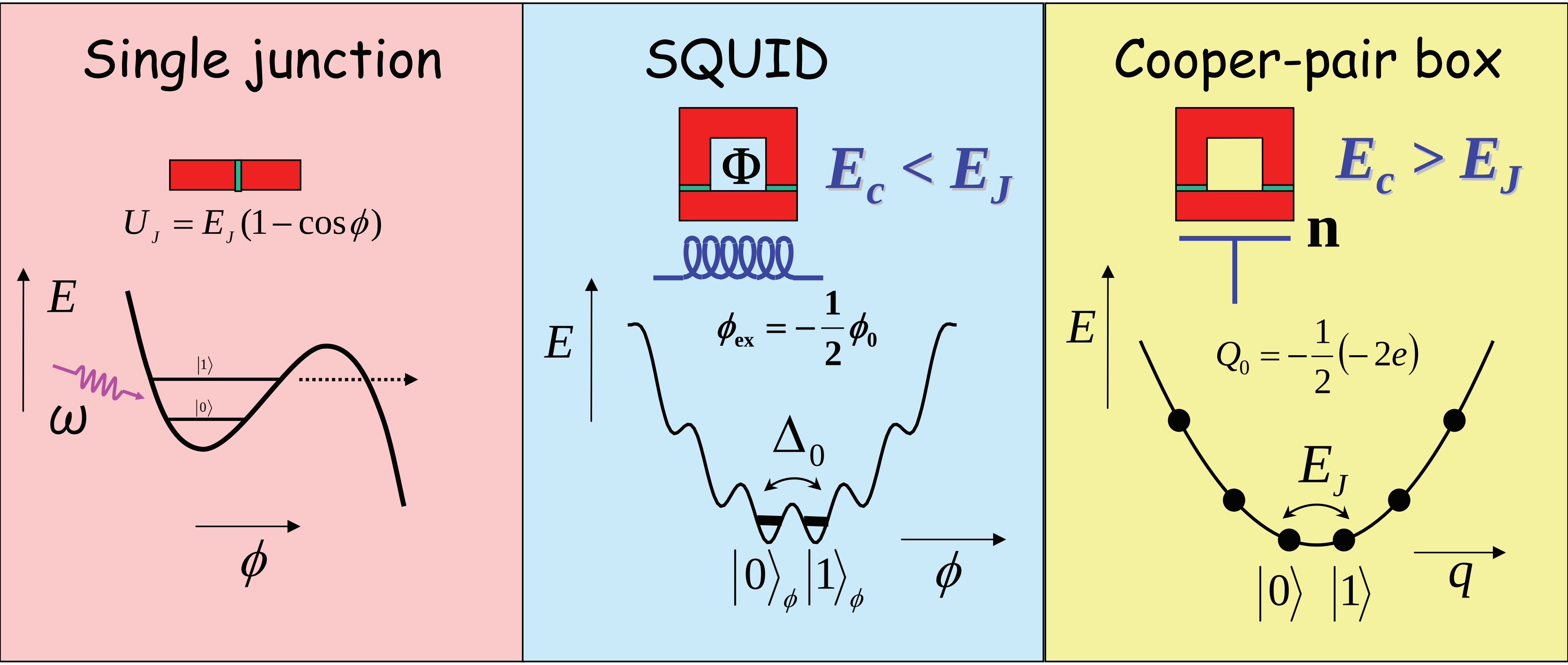
coupling easier →

Challenge: coupling vs. decoherence



Experimental challenge:
couple qubits to each other, control, & measure
not noise and dissipation

Josephson-junction-based qubits



NIST
Kansas
Maryland

Delft
NTT
Jena

Saclay

NEC
Chalmers
Yale
JPL

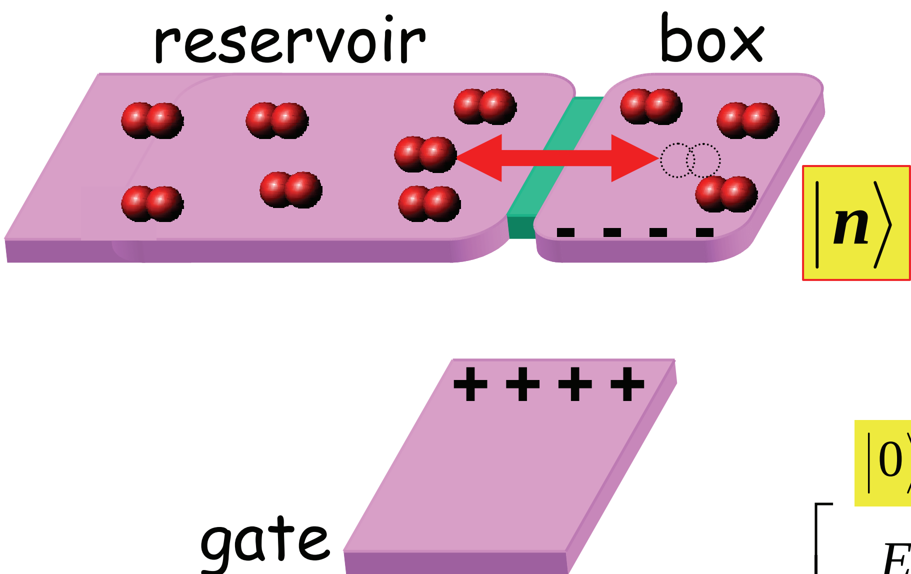
Cooper-pair box

M. Büttiker, 1987
V. Bouchiat et al, 1995

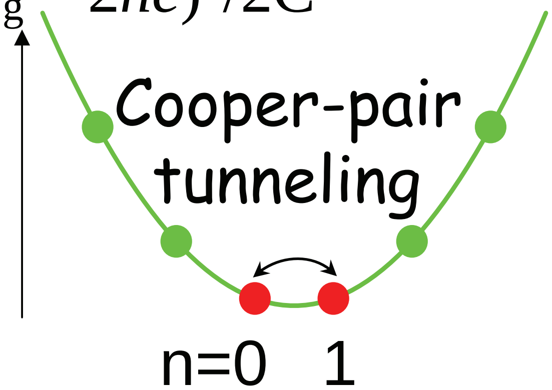
- a single artificial two-level system
- ~10^8 conduction electrons in the box

$\Delta > E_C$

$4E_C > E_J >> kT$



$E = (C_g V_g - 2ne)^2/2C$



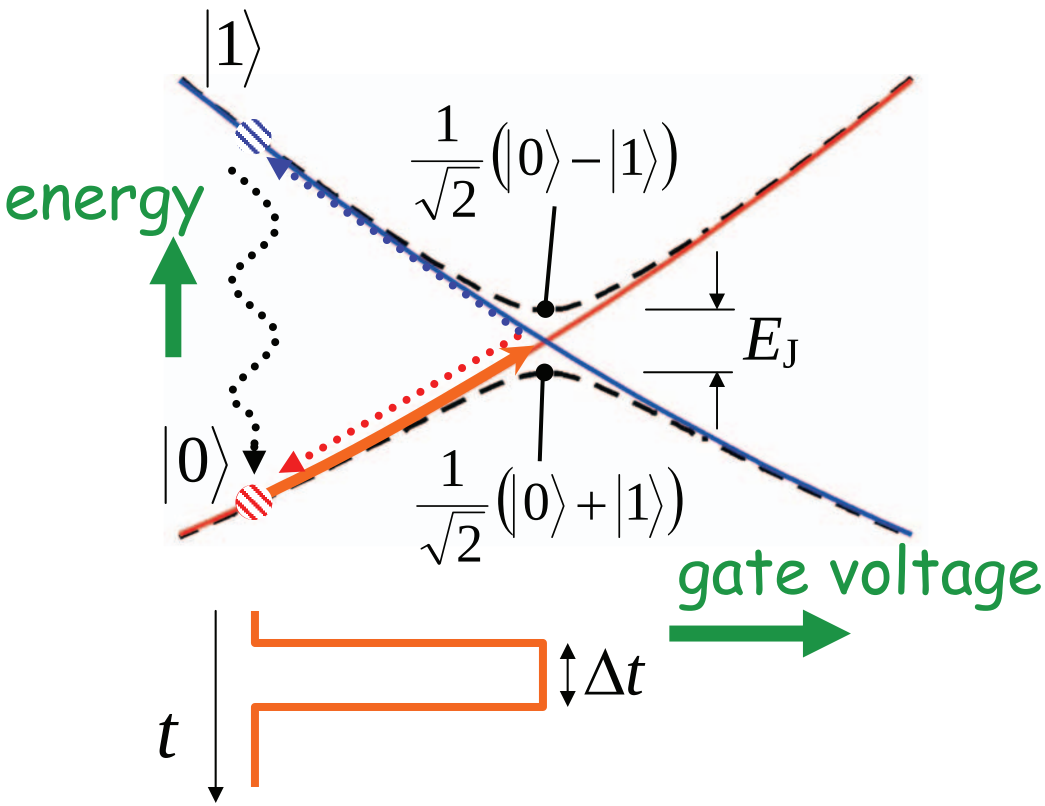
$$H = \begin{array}{cc} |0\rangle & |1\rangle \\ \left[\begin{array}{cc} E_0 & -\frac{1}{2}E_J \\ -\frac{1}{2}E_J & E_1 \end{array}\right] & \begin{array}{c} |0\rangle \\ |1\rangle \end{array} \end{array}$$

$$E_0 = E_c\left(0 - Q_g/2e\right)^2$$

$$E_1 = E_c\left(1 - Q_g/2e\right)^2$$

Charge qubit based on Cooper-pair box

Y. Nakamura et al, 1999



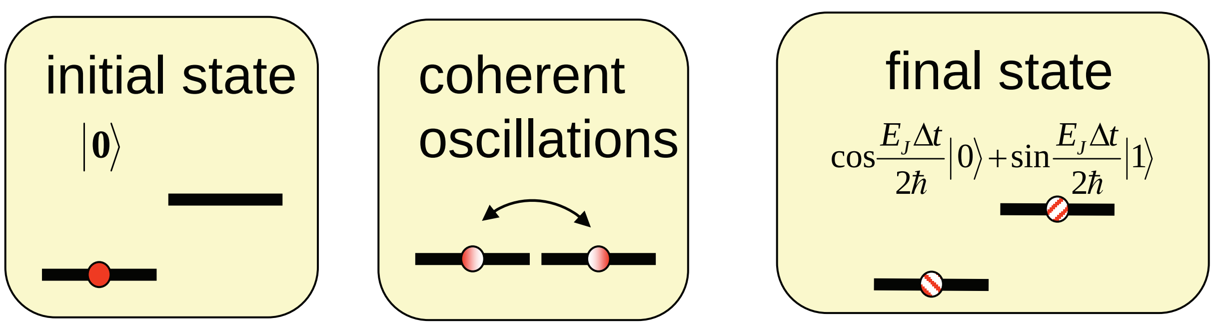
eigenstates: $|\psi_G\rangle, |\psi_E\rangle$

$$\Delta E(Q_0) = \sqrt{\delta E(Q_0)^2 + E_J{}^2}$$

charge states: $|0\rangle$, $|1\rangle$

$$\delta E(Q_0) \equiv E_c(Q_0/e - 1)$$

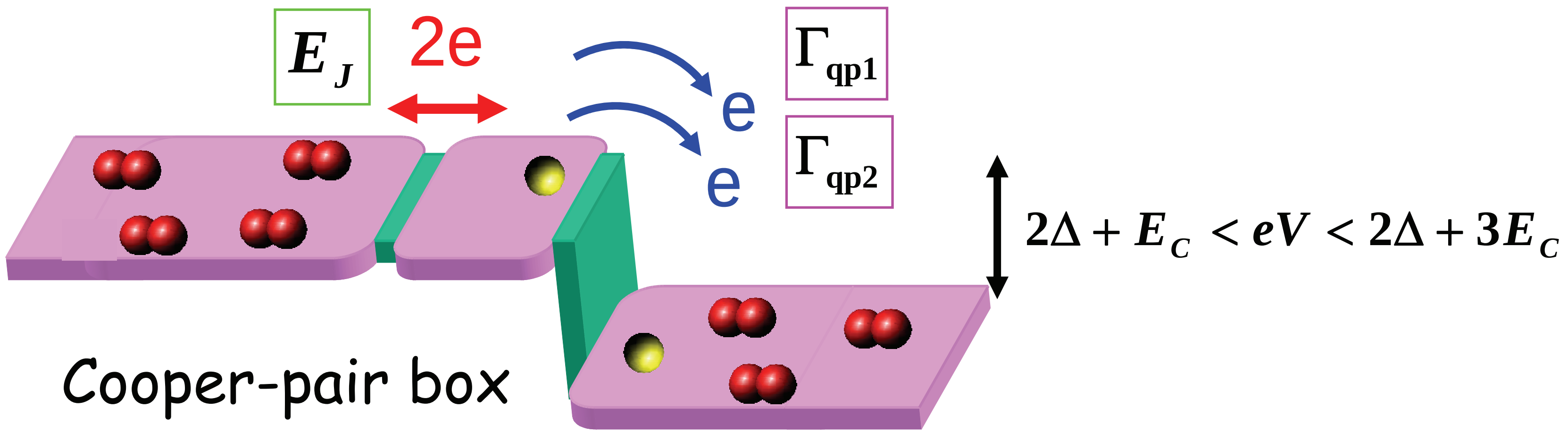
initialization
coherent superposition
read-out



$$\boxed{p(1) = \frac{1}{2}\left[1 - \cos\left(\frac{E_J \Delta t}{\hbar}\right)\right]}$$

Final state read-out

Josephson-quasiparticle cycle (Fulton et al., 1989)

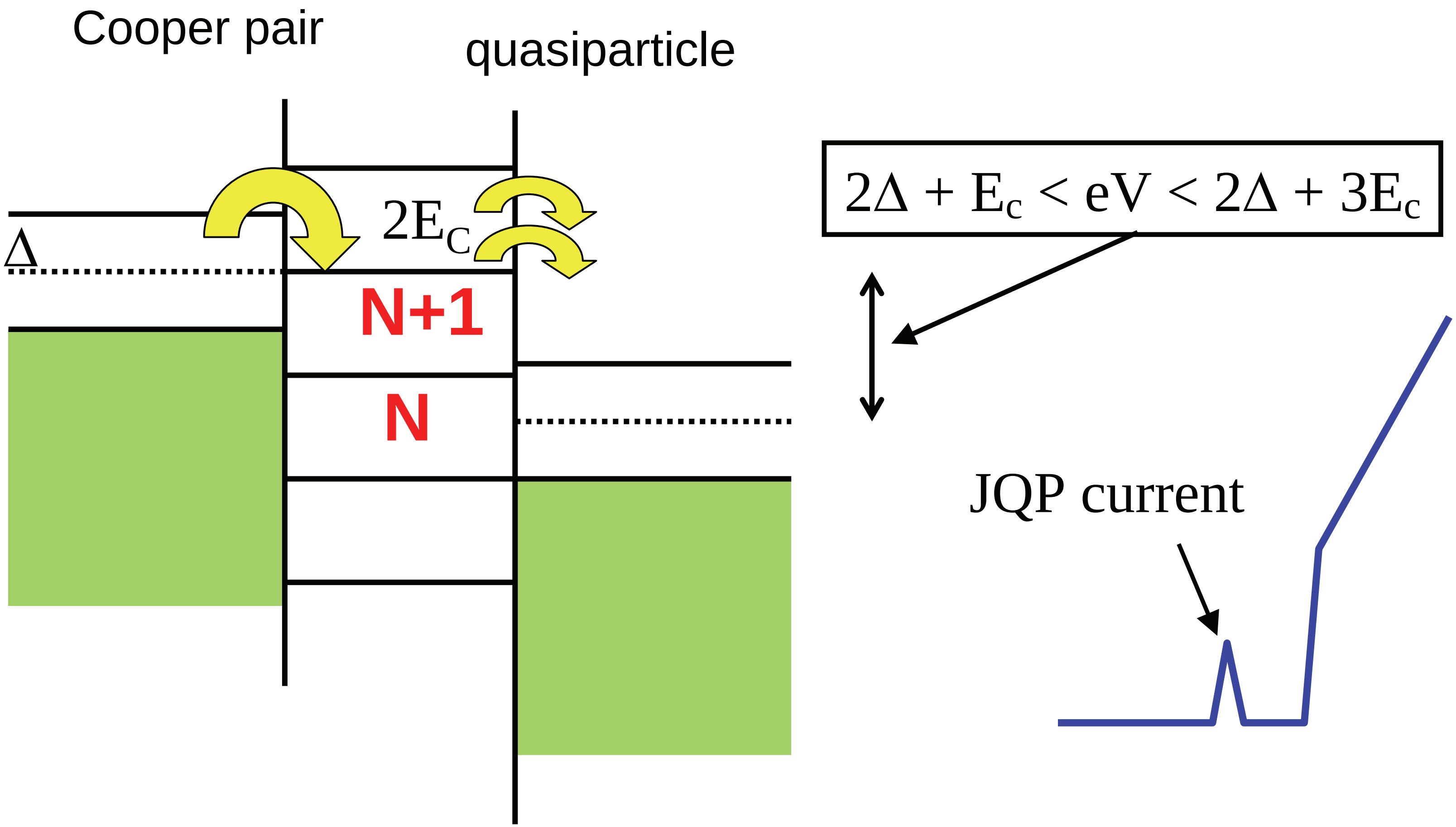


+ probe

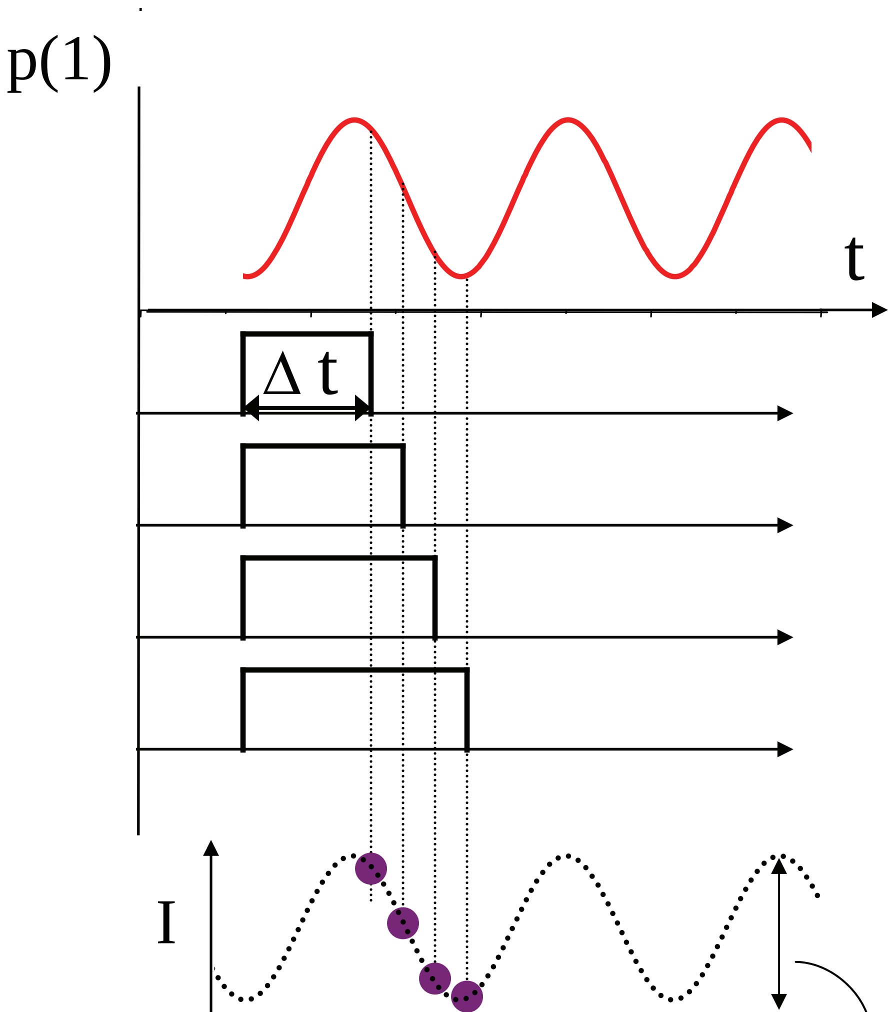
- detect the state $|1\rangle$
- initialize the system to $|0\rangle$

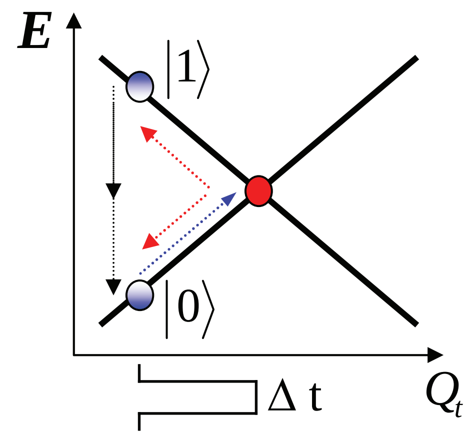
$$\boxed{E_J >> \hbar\Gamma_{qp1}}$$

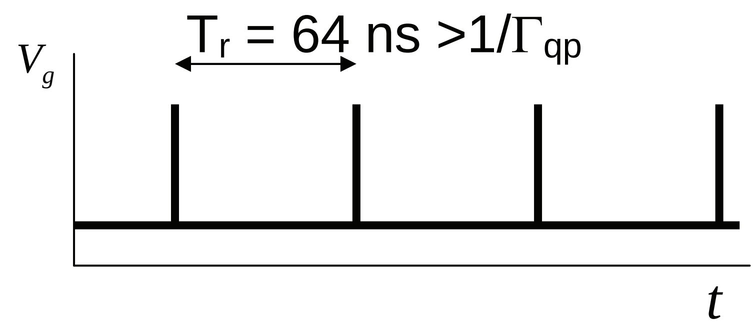
Josephson - quasiparticle (JQP) cycle



Tracing quantum oscillations: sampling technique





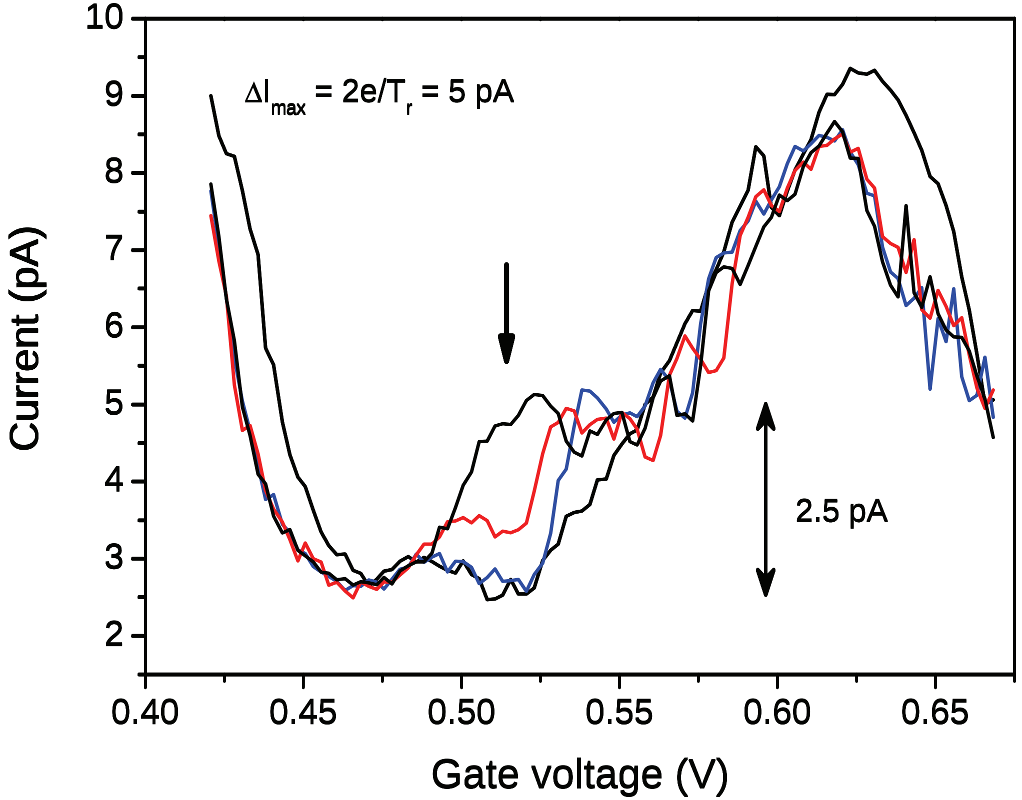


measurement time: 20 ms

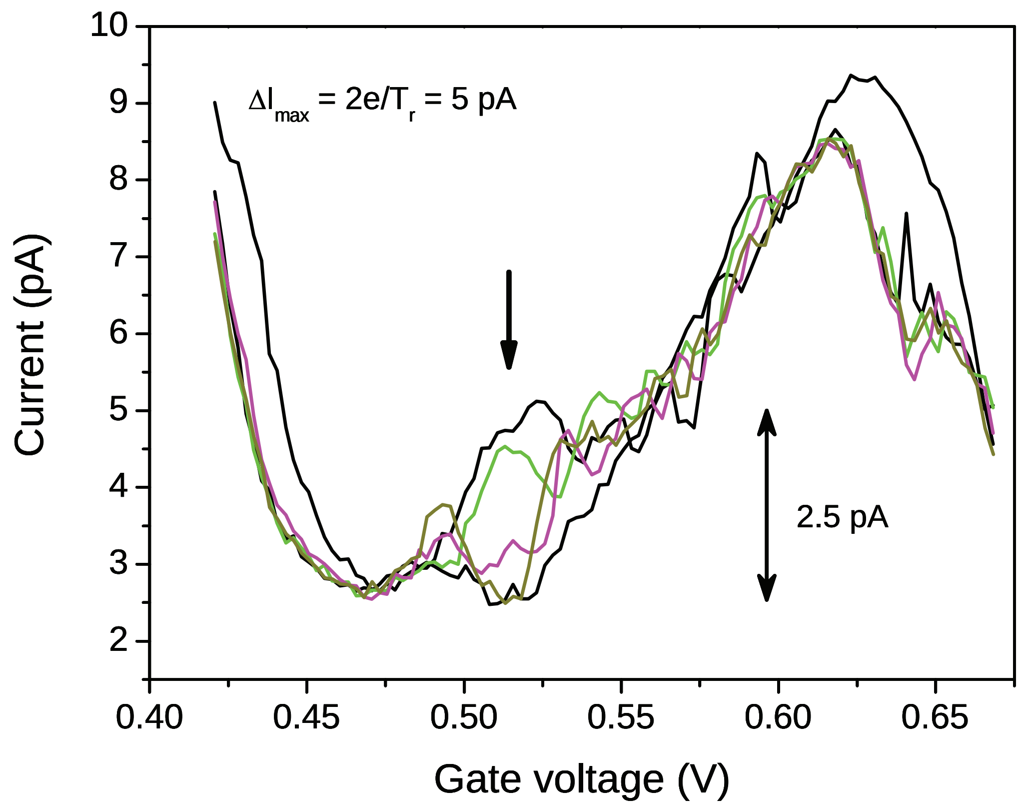
pulse array: 20ms/64 ns = 3×10^5 pulses

$$\boxed{I = 2e/T_r = 5 \text{ pA}}$$

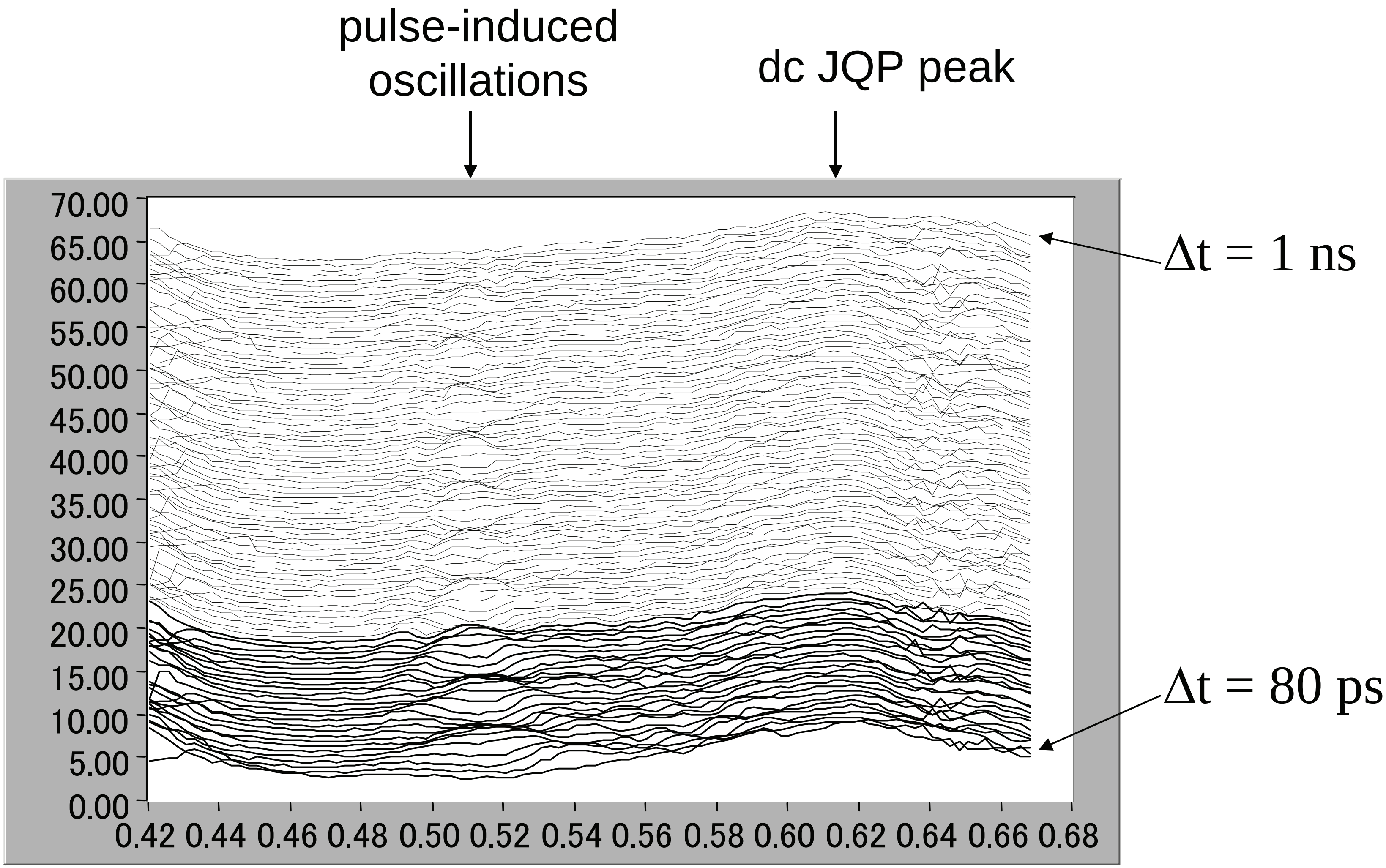
Dc sweep + pulses (1)

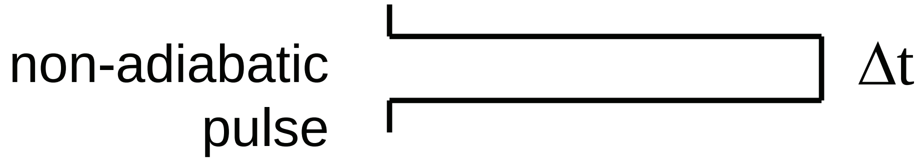


Dc sweep + pulses (2)

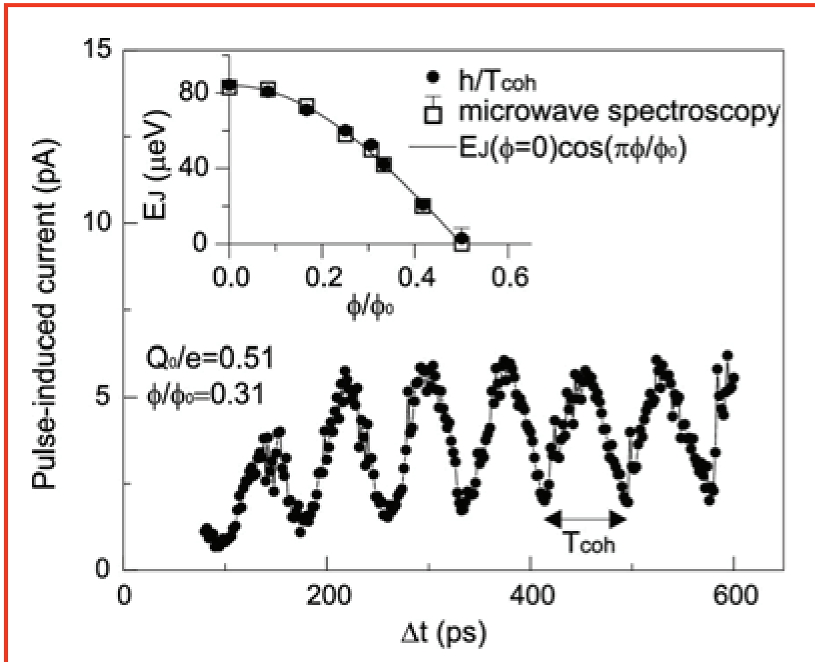


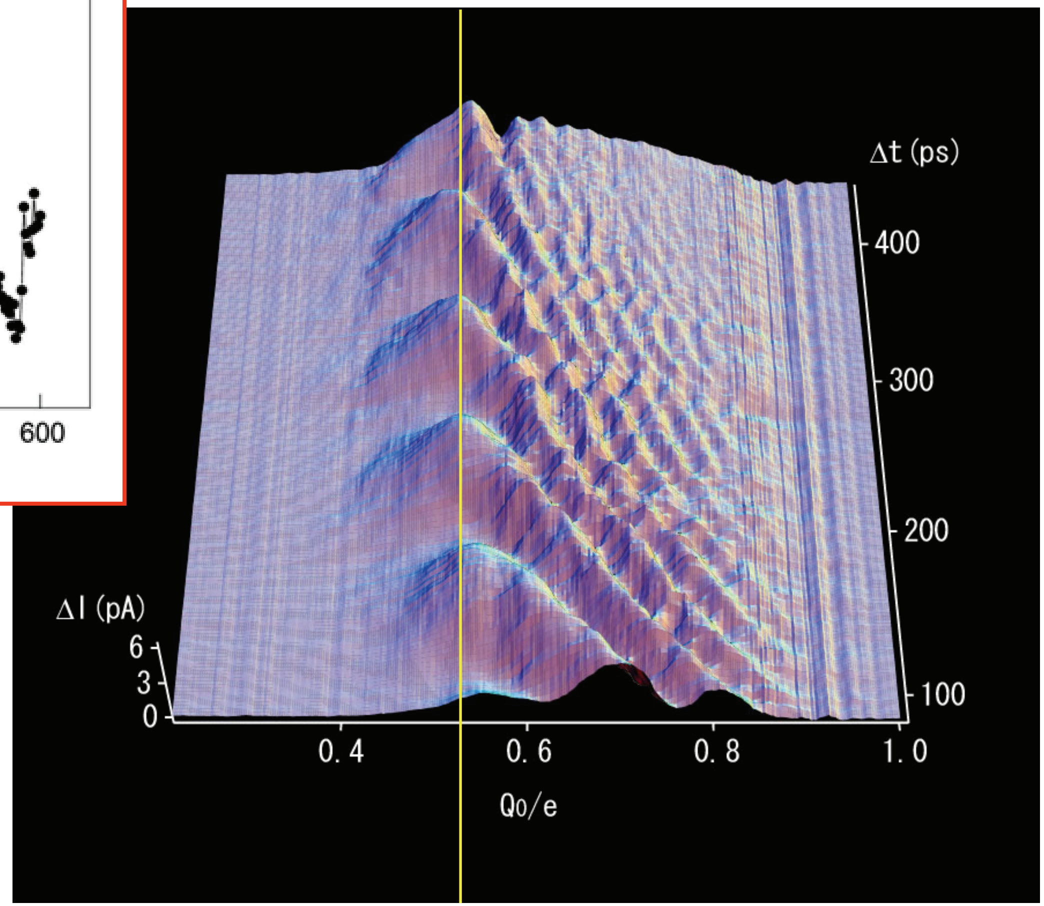
Dc sweep + pulses (4)





Coherent oscillations





Coherent control
of quantum state

Y. Nakamura *et al.*, Nature **398**, 786 (1999)

Coupling two charge qubits

E_1^1 E_0^1 qubit 1 $\overset{E_\mathrm{m}}{+}$ E_1^2 E_0^2 qubit 2 $=$ E_{11} E_{01} E_{10} E_{00} two-qubit states

coupling

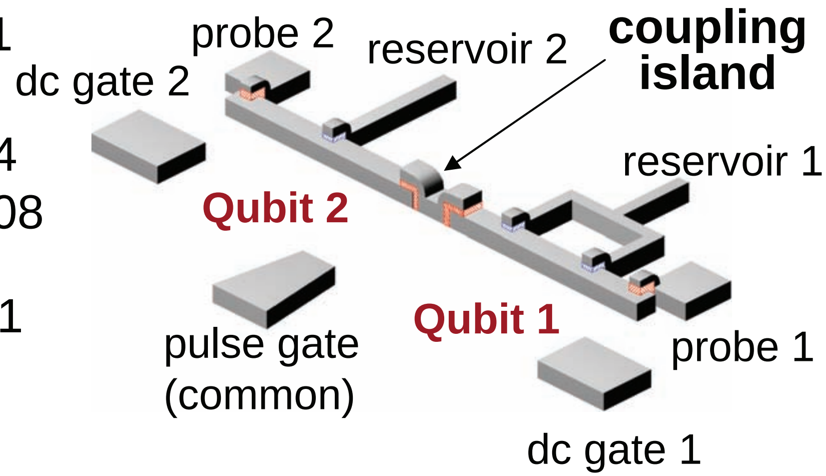
electrostatic ← our work

inductive - Yu. Makhlin et al. RMP 2001
J. Q. You et al. PRL 2002
J. Lantz et al. cond-mat 2004
T. Yamamoto et al. PRB 2008

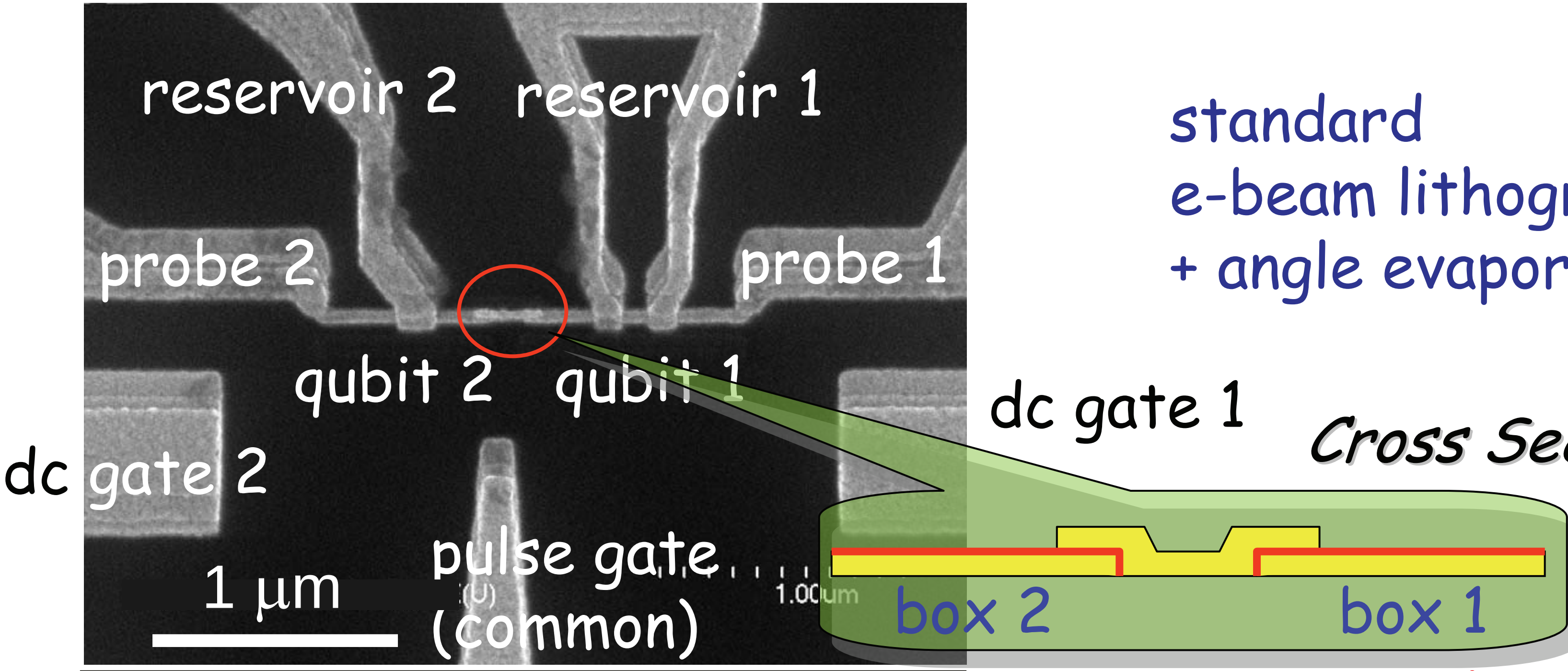
Josephson - F. Plastina et al. PRB 2001

for optimal mixing

$$\boxed{E_m \sim E_{J1}, E_{J2}}$$

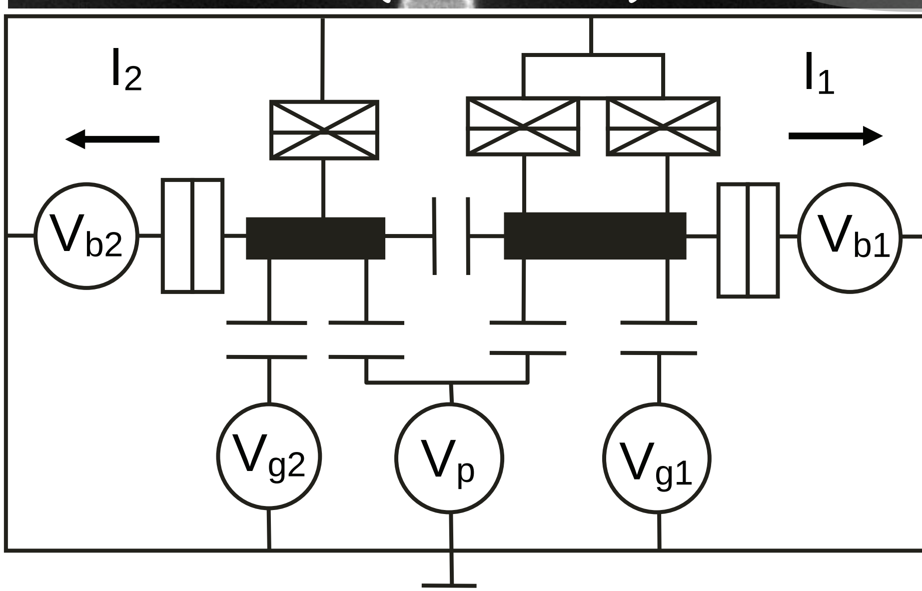


Capacitively-coupled charge qubits



standard
e-beam lithography
+ angle evaporation

capacitive coupling



I_1 and I_2 give info on charge states

Hamiltonian

charge basis

$$
\begin{array}{ccc}
 & \begin{array}{cccc} |00\rangle & |10\rangle & |01\rangle & |11\rangle \end{array} & \\
H = & \begin{bmatrix}
E_{00} & -\frac{1}{2}E_{J1} & -\frac{1}{2}E_{J2} & 0 \\
-\frac{1}{2}E_{J1} & E_{10} & 0 & -\frac{1}{2}E_{J2} \\
-\frac{1}{2}E_{J2} & 0 & E_{01} & -\frac{1}{2}E_{J1} \\
0 & -\frac{1}{2}E_{J2} & -\frac{1}{2}E_{J1} & E_{11}
\end{bmatrix} & \begin{array}{c} |00\rangle \\ |10\rangle \\ |01\rangle \\ |11\rangle \end{array}
\end{array}
$$

$$\boxed{\mathbf{E_{J1,2} \sim E_m < E_{c1,2}}}$$

initial state $|00\rangle$

E_{c1}, E_{c2}, E_m

E_{J1}, E_{J2}

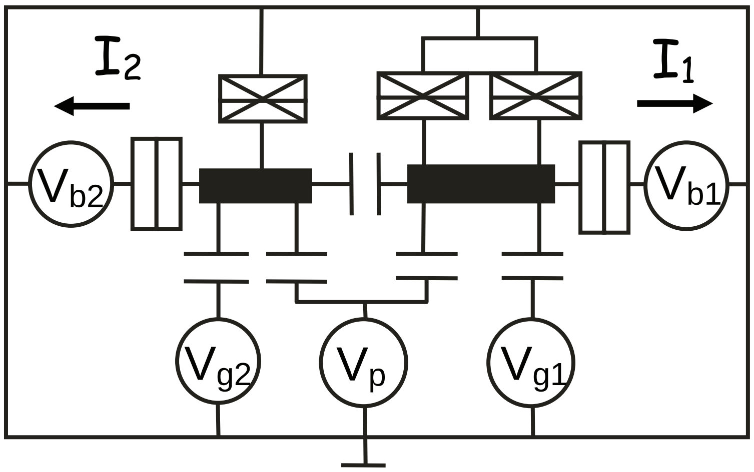
$$E_{n1n2} = E_{c1}(n_{g1}-n_1)^2 + E_{c2}(n_{g2}-n_2)^2 + E_m(n_{g1}-n_1)(n_{g2}-n_2)$$

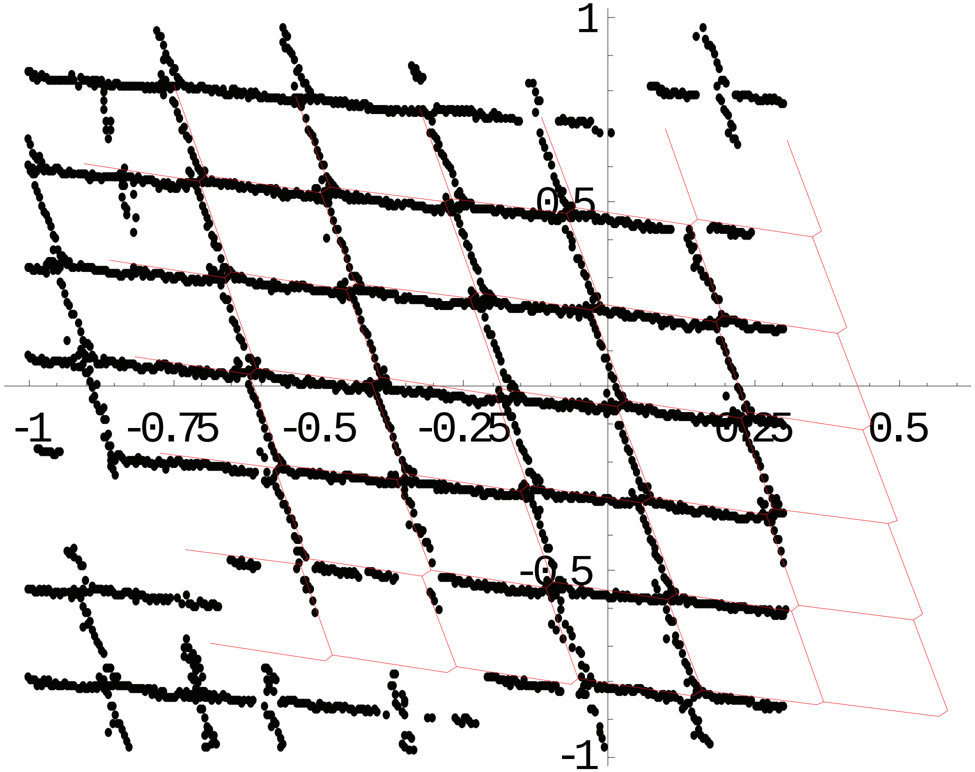
$$E_{c1,2} = 4e^2C_{\Sigma\,2,1}/2(C_{\Sigma\,1,2}C_{\Sigma\,2,1} - C_m^{\;2}) \approx 4e^2/2C_{\Sigma\,1,2}$$

$$n_{g1,2} = (C_{g1,2}V_{g1,2} + C_pV_p)/2e$$

$$E_m = 4e^2C_m/(C_{\Sigma\,1}C_{\Sigma\,2} - C_m^{\;2})$$

DC measurements

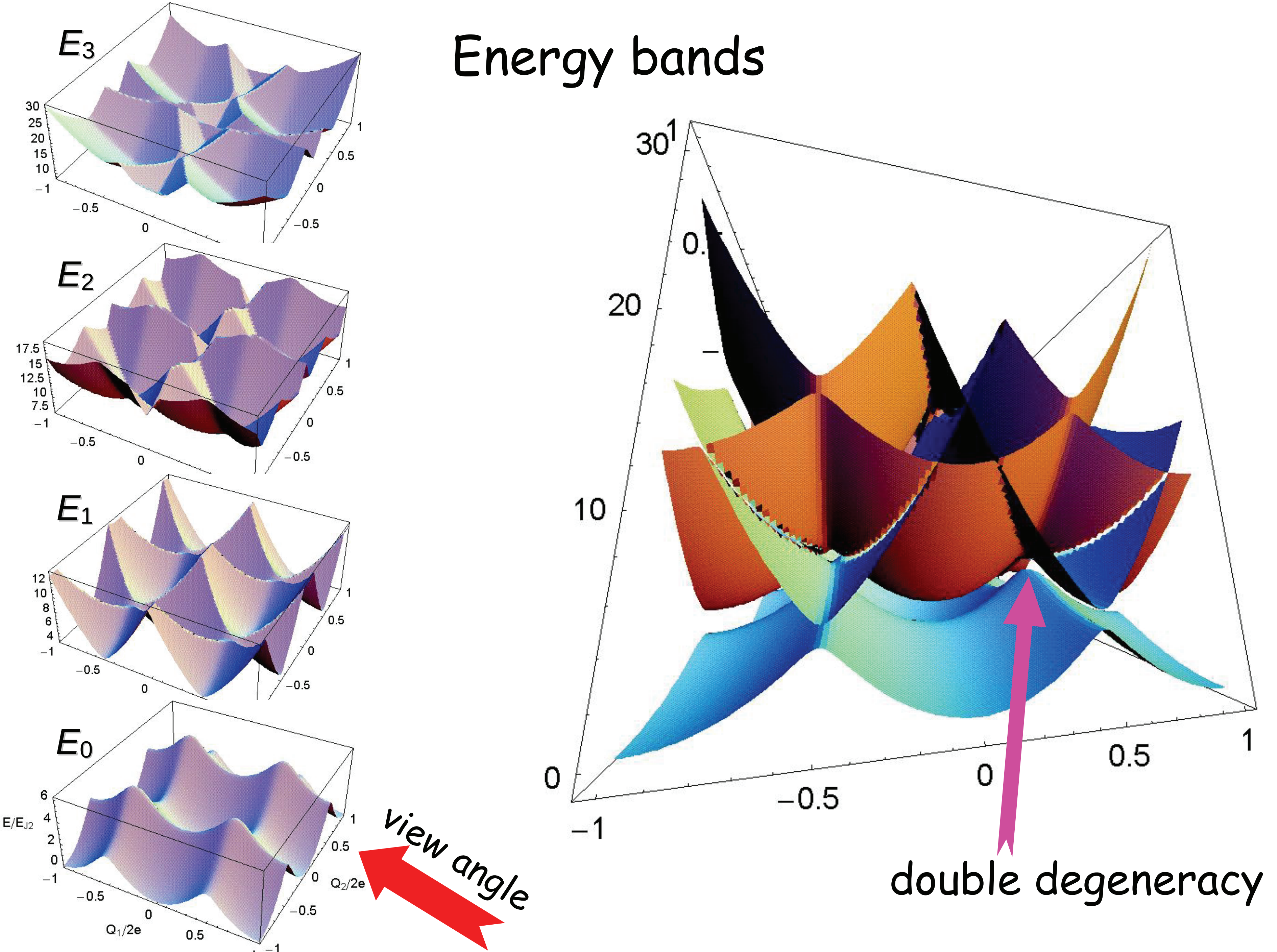




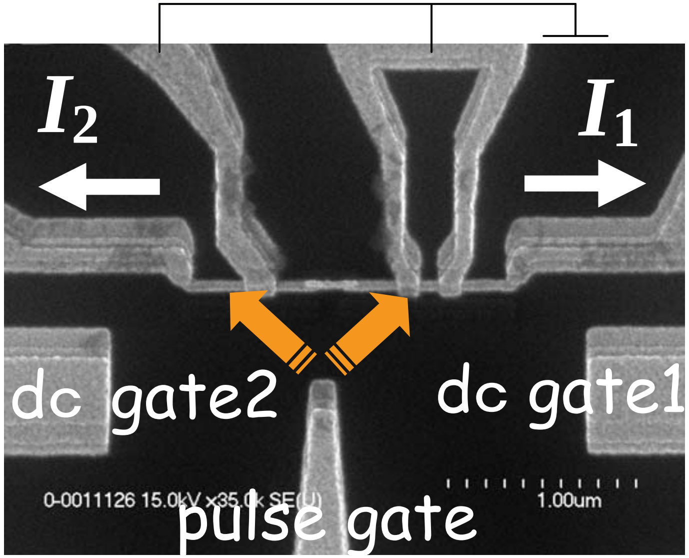
From the fit:

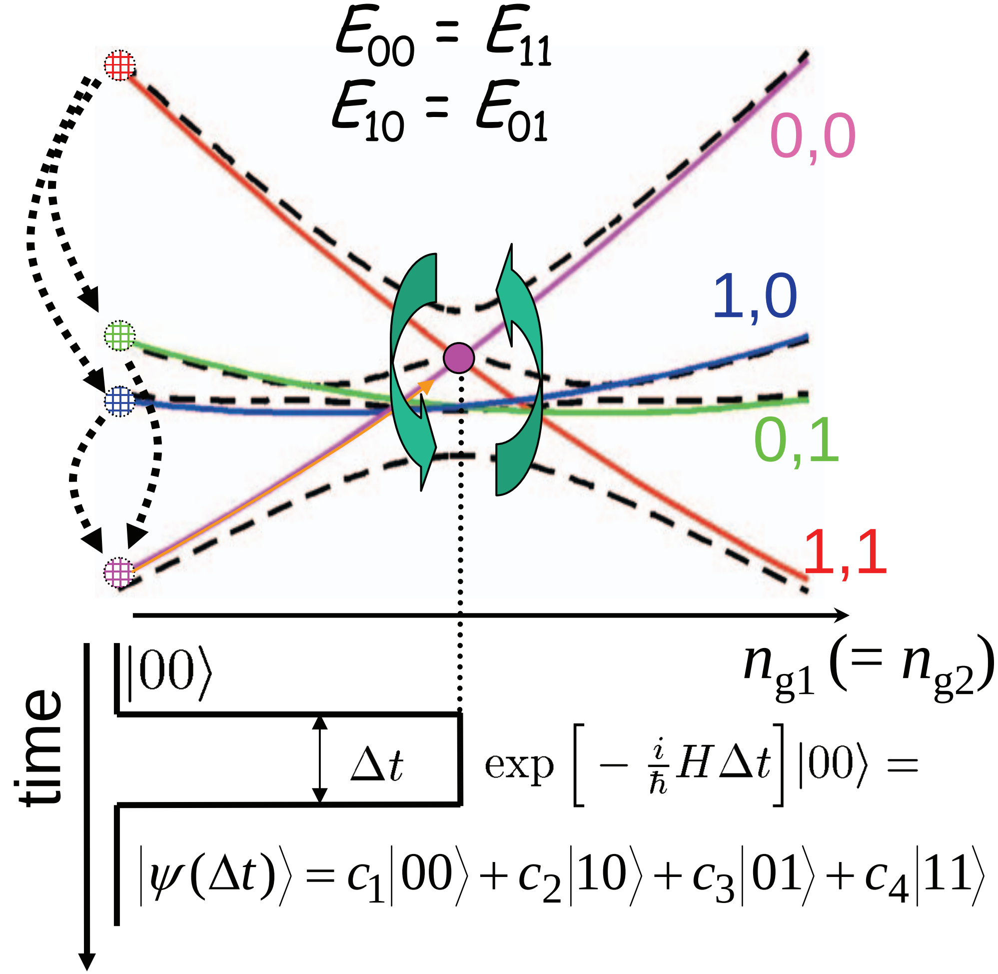
E_{c1} = 120 GHz
E_{c2} = 150 GHz
E_m = 15.7 GHz

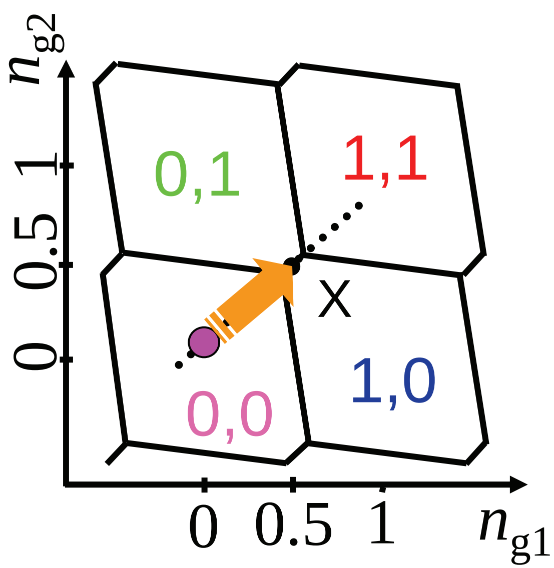
Energy bands



Oscillations at the double degeneracy







superposition of four charge states!

Quantum beatings

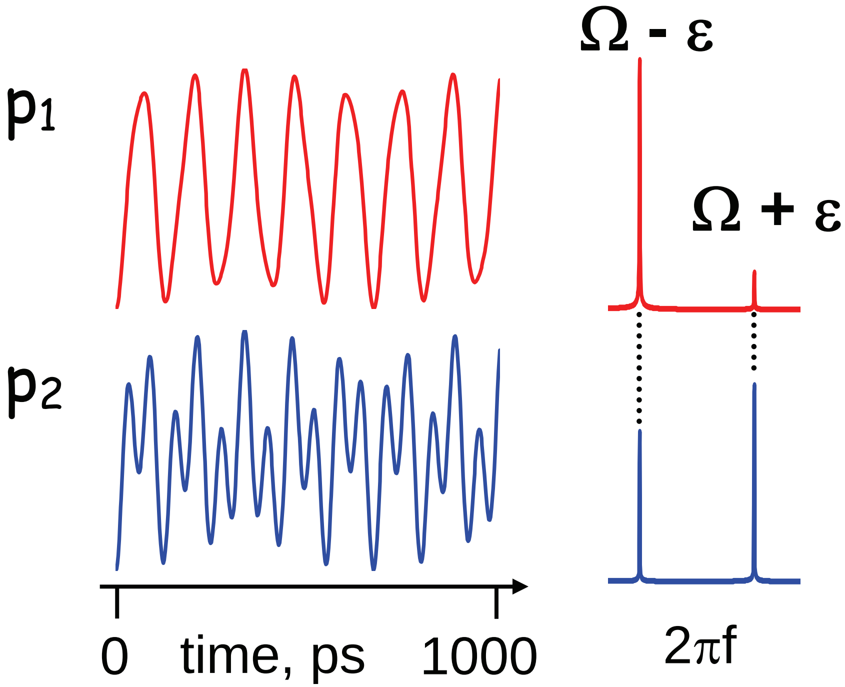
$$|\psi(t)\rangle = \exp\left[-\frac{i}{\hbar}Ht\right]|00\rangle$$

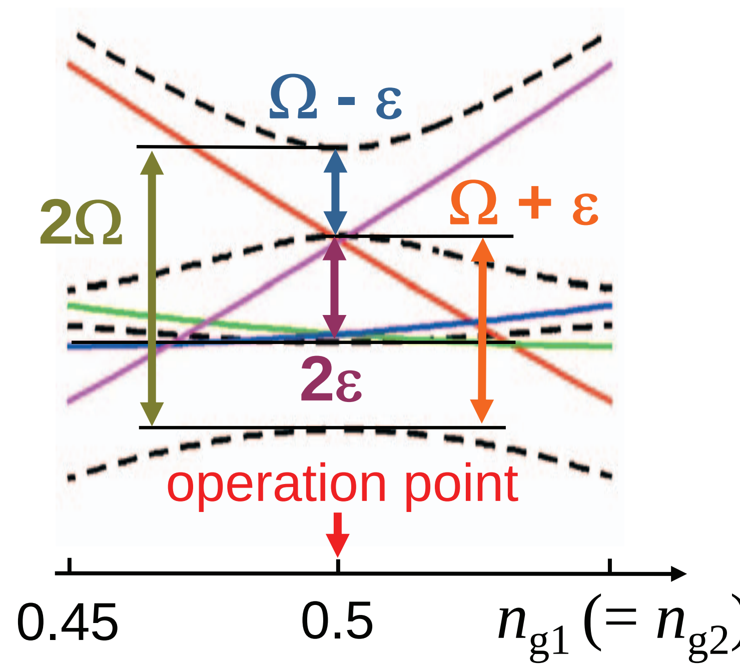
$$|\psi(t)\rangle = c_1|00\rangle + c_2|10\rangle + c_3|01\rangle + c_4|11\rangle$$

$$I_2 \propto p_2(1) \equiv |c_3|^2 + |c_4|^2 =$$

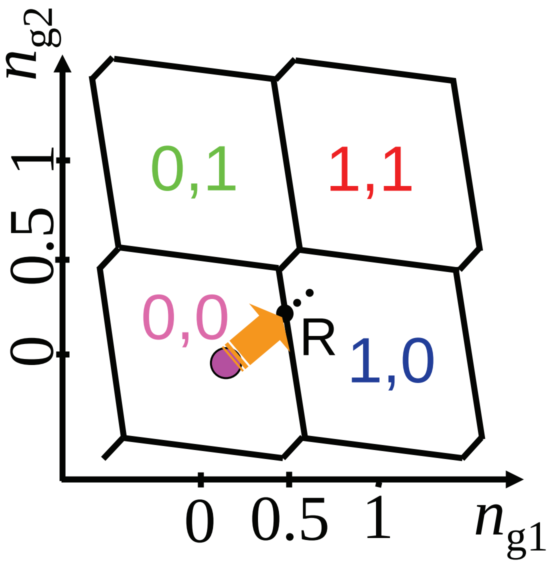
$$= \frac{1}{4}[2-(1-\chi)\cos(\Omega+\varepsilon)\Delta t) - (1+\chi)\cos(\Omega-\varepsilon)\Delta t)]$$

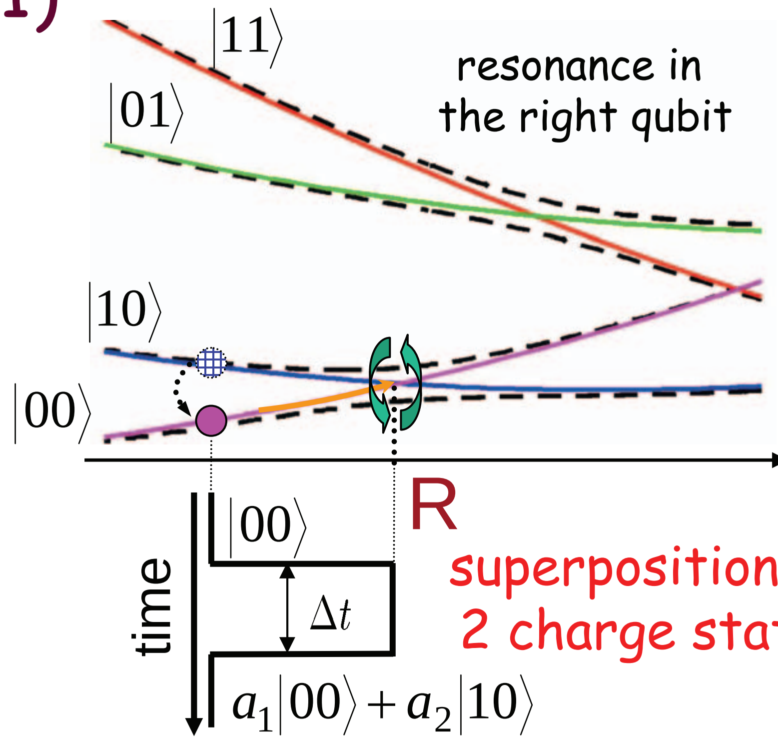
$$\begin{aligned} \chi &= \frac{E_{J1}^2 - E_{J2}^2 + (E_m/4)^2}{4\hbar^2\Omega\epsilon} \\ \Omega &= \sqrt{\Delta^2 + (E_m/4\hbar)^2} \\ \epsilon &= \sqrt{\delta^2 + (E_m/4\hbar)^2} \\ \Delta &= (E_{J2}+E_{J1})/2\hbar \\ \delta &= (E_{J2}-E_{J1})/2\hbar \end{aligned}$$





Independent oscillations (1)

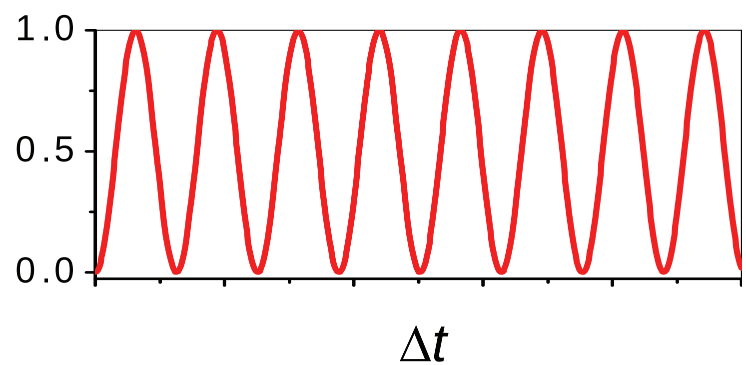




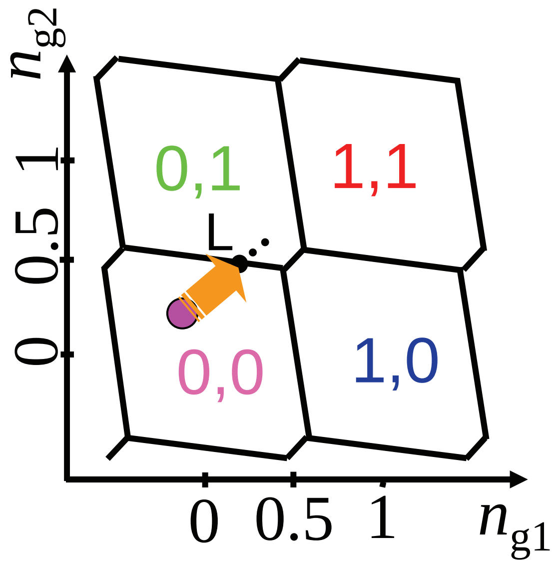
superposition of 2 charge states

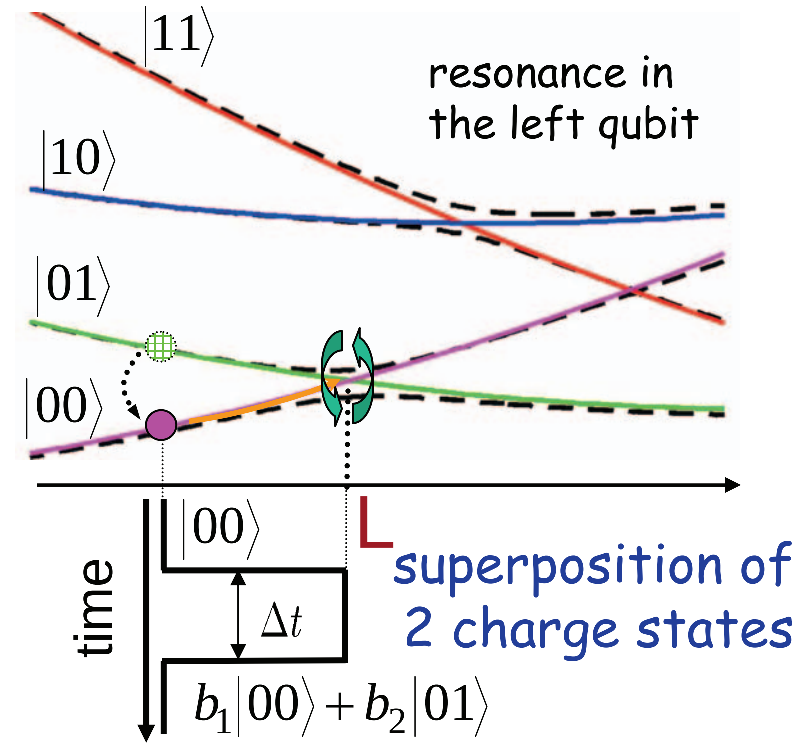
$$I_1 \propto p_1(1) \equiv |a_2|^2 = \frac{1}{2}\left[1 - \cos(\frac{E_{J1}}{\hbar}t)\right]$$

oscillations in qubit 1



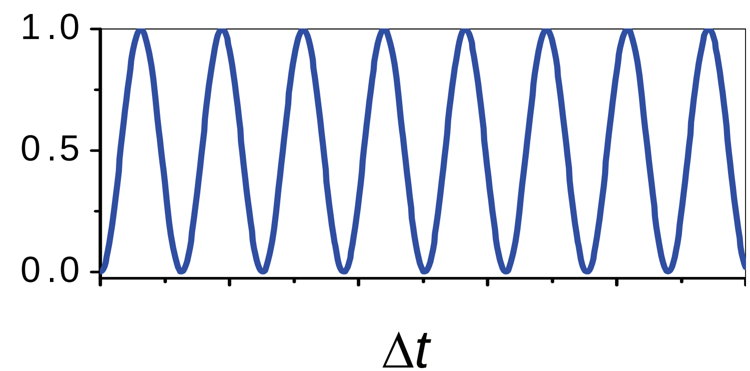
Independent oscillations (2)



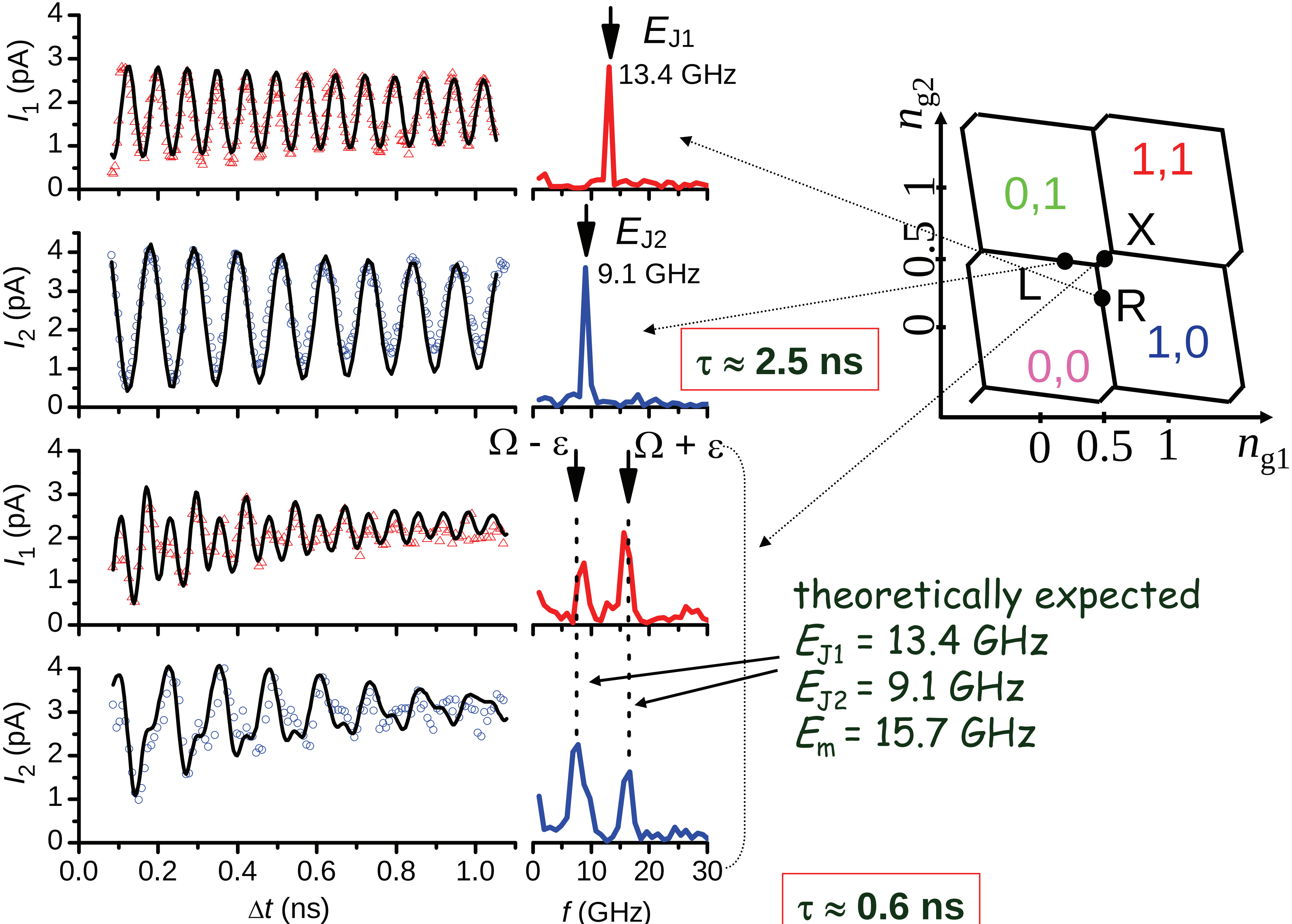


$$I_2 \propto p_2(1) \equiv |b_2|^2 = \frac{1}{2}\left[1 - \cos(\frac{E_{J2}}{\hbar}t)\right]$$

oscillations in qubit 2



Quantum beatings: experiment

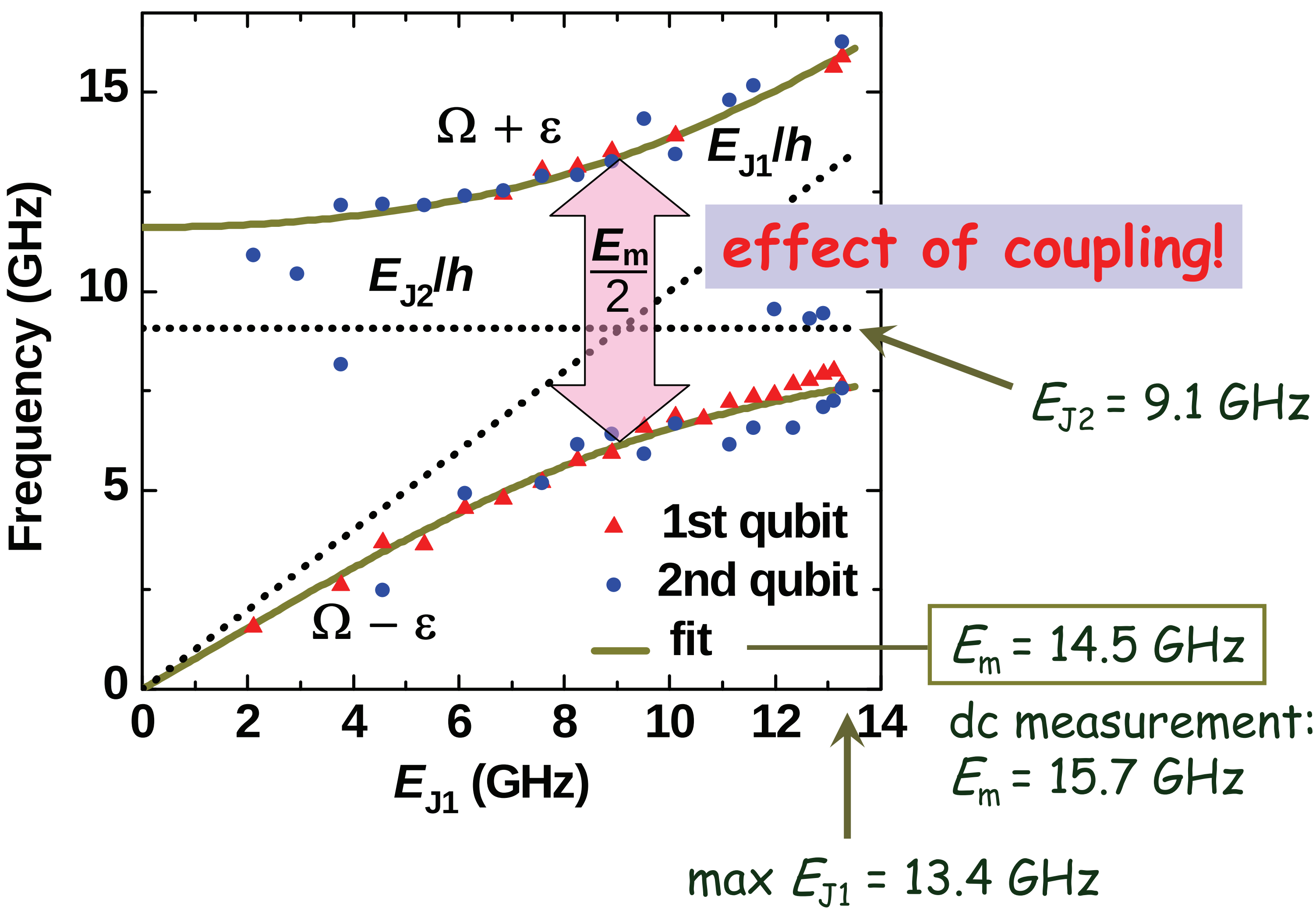


τ ≈ **2.5 ns**

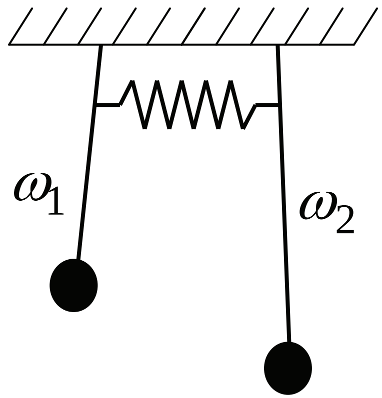
theoretically expected
E_{J1} = 13.4 GHz
E_{J2} = 9.1 GHz
E_m = 15.7 GHz

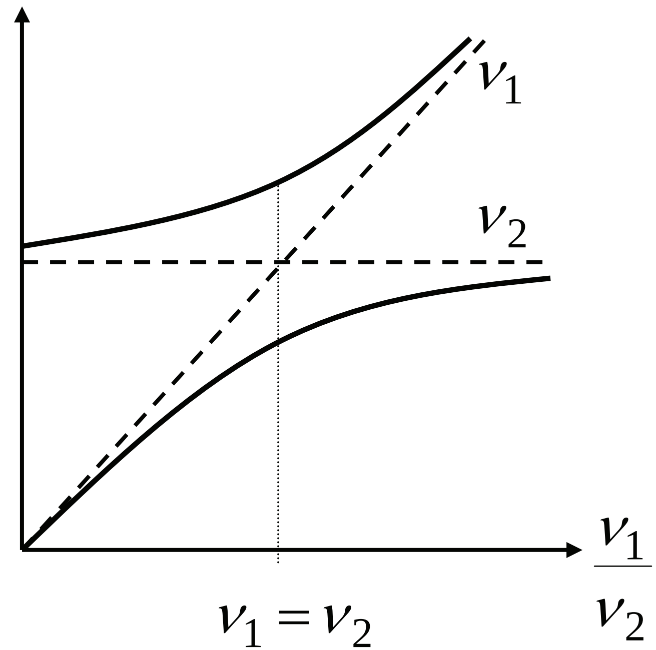
τ ≈ **0.6 ns**

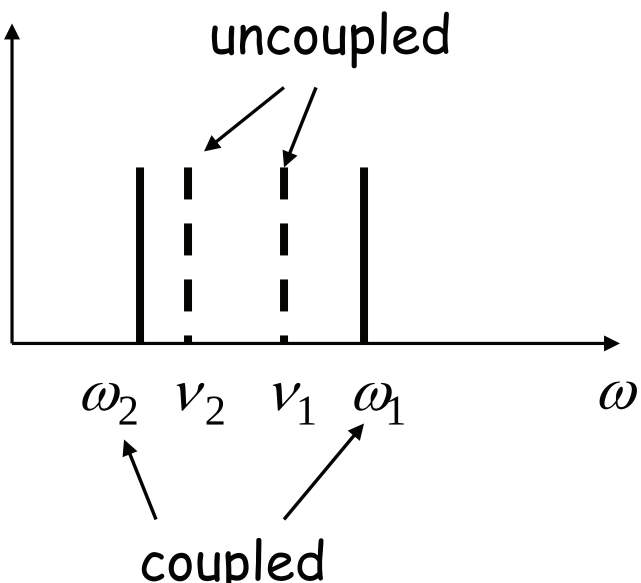
E_{J1} dependence of frequencies



Two coupled classical oscillators







What is the difference?

oscillates:

C.: physical parameter x

Q.: p(x) to be 0 or 1

Entanglement of two coupled qubits

Entangled qubits: $|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$

Entropy of entanglement:

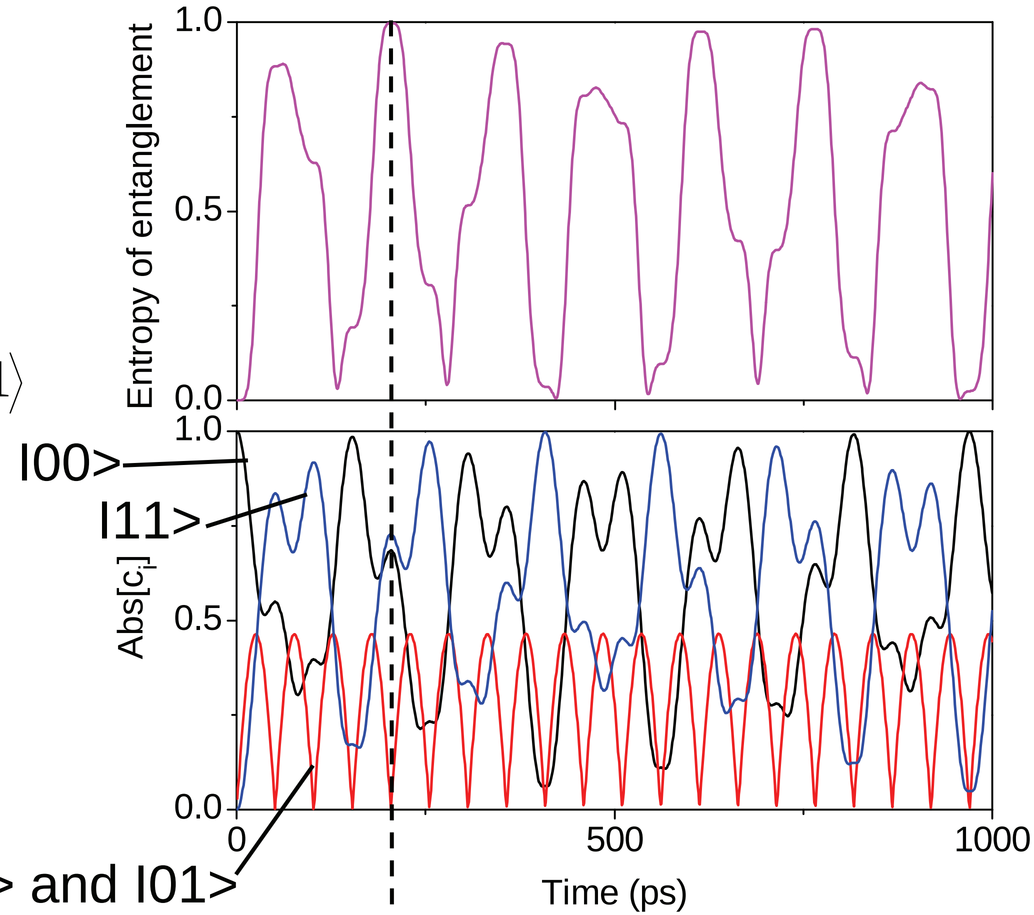
$$E = -Tr\rho_A \log_2 \rho_A = -Tr\rho_B \log_2 \rho_B$$

our qubits E_{J1} = 9.1 GHz
E_{J2} = 9.1 GHz
E_m = 14.5 GHz

If $|\psi\rangle = c_1|00\rangle + c_2|10\rangle + c_3|01\rangle + c_4|11\rangle$

$$\left.\begin{array}{l} |\psi\rangle = \dfrac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right) \\ |\psi\rangle = \dfrac{1}{\sqrt{2}}\left(|10\rangle \pm |01\rangle\right) \end{array}\right\}$$

maximally entangled states, E = 1



"almost" maximally entangled state

Status of Josephson charge qubits:

stage of proof of concepts passed

- ✓ • first solid-state qubit: coherent oscillations
- ✓ • 2 qubits coupled: beatings, avoided level crossing
- ✓ • conditional gate operation
- ✓ • single-shot readout
- ? • sources of decoherence